# Multi-result supercompilation as a tool for program analysis

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#### Program analysis by supercompilation

Let SC be a semantics-preserving supercompiler:

e' = SC[e] ⇒ e' ≅ e

The idea: instead of analyzing e, we may analyze SC[e].

- SC[e] may be easier to understand than e.
- Some hidden properties of e may become apparent in SC[e].

This is an instance of *transformational approach* to program analysis. (In principle, any semantics-preserving program transformer can be used.)

# An example: proving that all values returned by e satisfy p

Suppose that

- *e* is an expression.
- *p* is a predicate.
- e and p are written in the same language.
- SC is a semantics-preserving supercompiler.

How to prove that anything returned by *e* satisfies *p*? Consider the program *p e*, and supercompile it!

If we are lucky, SC[p e] is just (the constant) True. Thus,  $p e \cong SC[p e] = True$ .

# SC[*p e*] in action: the verification of protocols

- Alexei Lisitsa and Andrei P. Nemytykh. Reachability Analysis in Verification via Supercompilation. International Journal of Foundations of Computer Science, Vol. 19, No. 4 (2008) 953-969
- Alexei Lisitsa and Andrei P. Nemytykh. Verification as Specialization of Interpreters with Respect to Data. In Proceedings of First International Workshop on Metacomputation in Russia, META'2008. PereslavI-Zalessky, Russia, 2-5 July 2008, ISBN 978-5-901795-12-5, pp 94--112

SCP 4 : Verification of Protocols http://refal.botik.ru/protocols/

### Goals of supercompilation



- "Efficiency" can be measured (in bytes and seconds).
- "Understandability" is more difficult to measure and to formalize.

**A problem:** how to explain the goal to an analyzing SC? What is "good" and what is "bad"?

### First-order vs. higher-order SC

SCP4 (Nemytykh, <u>http://www.botik.ru/~scp/doc/docs.html</u>)

- Deals with Refal, a functional language.
- First-order.
- Call-by-value.
- Does not preserve termination properties of programs.

HOSC (Klyuchnikov, <a href="http://code.google.com/p/hosc/">http://code.google.com/p/hosc/</a>)

- Deals with HLL, a subset of Haskell.
- Higher-order
- Call-by-name.
- Preserves the semantics of programs.

## Why higher-order & call-by-name?

- A program is considered as a specification/formalization/model of something.
- A program is to be analyzed, rather than executed.

Higher-order:

- Specifications/models can be written in DSLs, implemented by combinators.
- Higher-order logics (quantifiers over functions/predicates).

Call-by-name:

- Termination properties are easier to preserve.
- Infinite data structures are useful for writing specifications of infinite processes.

### Higher-order SC: Church numbers (1)

Notation:  $f^{0} x = x$ ,  $f^{k} x = f(f^{k-1} x)$ .

data Nat = Z | S Nat;

Peano numbers:  $S^k Z$ . Church numbers:  $\s Z \rightarrow S^k Z$ .

unchurch = \n -> n S Z; foldn = \s z x -> case x of { Z -> z; S x1 -> s (foldn s z x1); };

church =  $\langle n - \rangle$  foldn ( $\langle m f x - \rangle f (m f x)$ )( $\langle f x - \rangle x$ ) n;

#### Higher-order SC: Church numbers (2)

add =  $\x y \rightarrow$  foldn S y x; mult =  $\x y \rightarrow$  foldn (add y) Z x;

churchAdd =  $\mbox{m} \rightarrow \mbox{n} \rightarrow \mbox{s} \rightarrow \mbox{z} \rightarrow \mbox{m} s$  (n s z); churchMult =  $\mbox{m} n f \rightarrow \mbox{m} (n f)$ ;

#### **A problem:** are the following expressions equivalent?

(mult x y)
unchurch(churchMult (church x) (church y))

## Higher-order SC: Church numbers (3)

By supercompiling the expressions with HOSC

```
(mult x y)
(unchurch(churchMult (church x) (church y))
```

we get the same residual program (modulo a renaming)!

```
letrec
f=(\s6->
   (\t6->
      case s6 of {
      Z -> Z;
      S u6 -> letrec g= \x7-> case x7 of {
           S v -> (S (g v));
           Z -> f u6 t6; }
           in (g t6);
      }))
in f x y
```

# Proving the equivalence of expressions by means of supercompilation (1)

Let  $\forall e, e \cong SC[e]$ , *i.e.* SC is semantics-preserving.

Proof technique:

 $SC[e_1] \equiv SC[e_2] \Leftrightarrow e_1 \cong e_2$ 

**Justification** (close to a tautology):

 $e_1 \cong SC[e_1] \equiv SC[e_2] \cong e_2$ 

**A problem**: SC may fail to guess, which versions of residual programs to produce, in order for them to be identical (modulo a renaming).

# Proving the equivalence of expressions by means of supercompilation (2)

 Alexei Lisitsa and Matt Webster. Supercompilation for Equivalence Testing in Metamorphic Computer Viruses Detection. In Proceedings of First International Workshop on Metacomputation in Russia, META'2008. PereslavI-Zalessky, Russia, 2-5 July 2008, ISBN 978-5-901795-12-5, pp 113-118.

Restrictions:

- All functions must be total (since SCP4 does not preserve termination properties).
- First-order logic (no quantifiers over functions/predicates, since Refal is a first-order language).
- No infinite data structures (Refal is a call-by-value language).

# Proving the equivalence of expressions by means of supercompilation (3)

 Ilya Klyuchnikov and Sergei Romanenko. Proving the Equivalence of Higher-Order Terms by Means of Supercompilation. In: Proceedings of the Seventh International Andrei Ershov Memorial Conference: Perspectives of System Informatics. LNCS 5947, 2009.

The technique was shown to work even if

- Functions may non-terminate.
- Free variables may be of functional types.
- Data may be infinite.

How? By *constructing* HOSC, a supercompiler that is really capable of "catching the mice".

# Another example: Proving the equivalence of small-step & big-step abstract machines

 Olivier Danvy and Kevin Millikin. 2008. On the equivalence between small-step and big-step abstract machines: a simple application of lightweight fusion. *Inf. Process. Lett.* 106, 3 (April 2008), 100-109. <u>PDF http://dx.doi.org/10.1016/j.ipl.2007.10.010</u>

An example of *transformational approach* to program analysis.

A **manual** proof by Danvy & Millikin: a non-trivial sequence of program transformations from the first program to the second one.

A proof **by supercompilation**: just supercompile two programs by HOSC to get the same residual program! (See <u>live</u>.)

# Tuning SC for program analysis (1)

 Ilya Klyuchnikov. Supercompiler HOSC 1.0: under the hood. Preprint 63. Keldysh Institute of Applied Mathematics, Moscow, 2009. <u>http://pat.keldysh.ru/~ilya/</u>

Gives a refined definition of homeomorphic embedding taking into account the difference between free and bound variables.

HOSC 1.0 *does not terminate* for some input programs (<u>an</u> <u>example</u>)!

• Ilya Klyuchnikov. Supercompiler HOSC 1.1: proof of termination. Preprint 21. Keldysh Institute of Applied Mathematics, Moscow, 2010.

HOSC 1.1 always terminates.

#### Extended homeomorphic embedding

#### **Classic embedding**

VariablesDivingCoupling $v_1 \leq v_2$  $\exists i : e \leq e'_i$  $\forall i : e_i \leq e'_i$  $v_1 \leq v_2$  $e \leq \phi(e'_1, \dots, e'_k)$  $\forall e_1, \dots, e_k \leq \phi(e'_1, \dots, e'_k)$ 

#### Extended embedding

 $e' \trianglelefteq e'' \mid_{\rho} \quad \text{ if } e' \trianglelefteq_v e'' \mid_{\rho} \text{ or } e' \trianglelefteq_d e'' \mid_{\rho} \text{ or } e' \trianglelefteq_c e'' \mid_{\rho}$ 

#### Variables

Diving

#### Coupling

$$\begin{array}{ll} c \ \overline{e'_i} \ \trianglelefteq_c \ c \ \overline{e''_i} \ |_{\rho} & \text{if } \forall i : e'_i \ \trianglelefteq \ e''_i \ |_{\rho} \\ \lambda v_1 \to e_1 \ \trianglelefteq_c \ \lambda v_2 \to e_2 \ |_{\rho} & \text{if } e_1 \ \trianglelefteq \ e_1 \ |_{\rho \cup \{(v_1, v_2)\}} \\ e' \ \overline{e'_i} \ \trianglelefteq_c \ e'' \ \overline{e''_i} \ |_{\rho} & \text{if } e' \ \trianglelefteq \ e''_i \ |_{\rho} \\ case \ e' \ of \ \{\overline{c_i \ \overline{v'_{ik}} \to e'_i;}\} \ \trianglelefteq_c \ case \ e'' \ of \ \{\overline{c_i \ \overline{v''_{ik}} \to e''_i;}\} |_{\rho} \\ & \text{if } e' \ \trianglelefteq \ e''_i |_{\rho} \ \text{and} \ \forall i : e'_i \ \trianglelefteq \ e''_i |_{\rho \cup \{\overline{(v'_{ik}, v''_{ik})}\}} \end{array}$$

#### Extended embedding is well-quasiorder

**Theorem (Kruskal, Higman)**. For any infinite sequence of expressions  $e_1, e_2, \dots e_n, \dots$ there are i<j, such that  $e_i \triangleleft e_j$ 

Extended whistle doesn't blow for ANY sequence!

**Theorem (Klyuchnikov)**. For any infinite sequence of expressions  $e_1, e_2, ..., e_n, ...$ , appearing on *a branch of partial process tree* t, there are i<j, such that  $e_i \triangleleft^* e_j$ 

# The proof of the pudding is in the eating

HOSC 0, using the "classic" homeomorphic embedding, proved only 6 of 25 equivalences (from the first chapter).

HOSC 1, using "extended" homeomorphic embedding, is able to prove 25 of 25 equivalences.



# Tuning SC for program analysis (2)

 Ilya Klyuchnikov. Supercompiler HOSC 1.5: homeomorphic embedding and generalization in a higher-order setting.
 Preprint 62. Keldysh Institute of Applied Mathematics, Moscow, 2010. <u>http://pat.keldysh.ru/~ilya/</u>

HOSC 1.5 is a revised (and simplified) version of HOSC 1.1.

The first published algorithm for finding a most specific generalization for expressions *with bound variables*.

The definition of homeomorphic embedding should take into account the possibility of the following *generalization*!

```
Unlike x and S x,
there is no embedding for \lambda x.x and \lambda x.S x !
```

#### "Higher-level" supercompilation?

An idea:

- The power of "ordinary" ("basic"?, "ground"?) is limited.
- Let us consider supercompilation as a "primitive operation" and construct a "metasystem" (in V.F. Turchin's terms).

The "ground" supercompilers are controlled by the metalevel (which, eventually, may be a supercompiler as well).



#### Examples of higher-level SC

- Futamura projections:  $SC_3[SC_2[SC_1]]$ .
  - $\circ$  SC<sub>2</sub> simulates the execution of SC<sub>1</sub>, and this is controlled by SC<sub>3</sub>.
- Proving the equivalence of expressions.

 $\circ$  SC[ $e_1$ ] ≡ SC[ $e_2$ ].

- Proving improvement lemmas  $(e_1, e_2)$ .
  - $SC[e_1] \equiv SC[e_2]$  and tick annotations in  $SC[e_1]$  are embedded in tick annotations in  $SC[e_2]$ .
- Two-level supercompilation.
  - The "upper" supercompiler applies improvement lemmas checked by means of the "ground" supercompiler.
- Distillation (Hamilton, ...).

# Checking improvement lemmas (1)

 Ilya Klyuchnikov and Sergei Romanenko. Towards Higher-Level Supercompilation. In Second International Workshop on Metacomputation in Russia (Proceedings of the second International Workshop on Metacomputation in Russia. Pereslavl-Zalessky, Russia, July 1-5, 2010). A. P. Nemytykh, Ed. - Pereslavl-Zalessky: Ailamazyan University of Pereslavl, 2010, 186 p. ISBN 978-5-901795-21-7, pages 82-101. <u>http://pat.keldysh.ru/~ilya/</u>

The idea:

- Mark in the residual program with "ticks" (the points where there has been an unfolding step during driving).
- Check two residual expressions for "homeomorphic embedding" with respect to "ticks".

#### Checking improvement lemmas (2)

Annotating a partial process tree with "ticks"



Propagating "ticks" into the residual program

letrec f=\*\v  $\rightarrow$  case v of { Z  $\rightarrow$  True; S p  $\rightarrow$  \*case p of { Z $\rightarrow$  (letrec g = \*\w  $\rightarrow$  case w of { Z  $\rightarrow$  False; S t  $\rightarrow$  \* case t of {Z  $\rightarrow$  True; S z  $\rightarrow$  g z;};} in g n; S x  $\rightarrow$  f x;}; in f n

## Checking improvement lemmas (3)

 Ilya Klyuchnikov. Towards effective two-level supercompilation. Preprint 81. Keldysh Institute of Applied Mathematics, Moscow. 2010. <u>http://pat.keldysh.ru/~ilya/</u>

What is new:

- An explicit algorithm for generating the residual program annotated with ticks from a partial process tree.
- An improved algorithm for comparing tick annotations based on normalization of ticks.

## Two-level supercompilation (1)

The goal is to avoid generalization!



- The whistle blows for  $\alpha$  and  $\beta$ .
- $\alpha$  (or  $\beta$ ?) has to be generalized.
- A generalization is an evil, as it causes some loss of information.

## Two-level supercompilation (2)

The goal is to avoid generalization!



- Let γ be an expression such that
  - $\circ$  ( $\beta$ ,  $\gamma$ ) is an improvement lemma;
  - $\circ$  the whistle is silent for  $\alpha$  and  $\gamma.$
- By replacing  $\beta$  with  $\gamma$ , we can avoid generalization!

#### Two-level supercompilation (3)

```
...
if whistle(e1, e2)
abstract(e1, e2)
...
}
scp1 is simplified (no
```

def scp $0(e) = \{$ 

check for improvement).

```
def scp1(e) = {
```

```
if whistle(e1, e2)
 e3 = findEquiv(e1)
 if e3 != null
  replace(e1, e3)
 else
  abstract(e1, e2)
def findEquiv(e1) = {
 for c <- candidates(e1)
  if scp0(e1) == scp0(c)
    return c
 return null
}
```

## Two-level supercompilation (4)

How to speed up the search for lemmas and make the lemmas "friendlier"?

 Ilya Klyuchnikov. Towards effective two-level supercompilation. Preprint 81. Keldysh Institute of Applied Mathematics, Moscow. 2010. <u>http://pat.keldysh.ru/~ilya/</u>

Some tricks related to

- finding improvement lemmas by inspecting and manipulating the expressions that have already appeared in the partial process tree;
- extracting "human-friendly" (and more abstract) lemmas from the lemmas produced automatically (which are often cumbersome and too specific).

## Two-level supercompilation (5)

**Theorem** (Sørensen, 1994). Classical positive supercompiler for a call-by-name language cannot improve the asymptotic of a program.

However, as shown in

 Ilya Klyuchnikov. Towards effective two-level supercompilation. Preprint 81. Keldysh Institute of Applied Mathematics, Moscow. 2010. <u>http://pat.keldysh.ru/~ilya/</u>

an  $O(n^2)$  parser corresponding to the grammar p = a p a | empty.

can be transformed by a 2-level supercompiler to an O(n) parser corresponding to the grammar:

p' = a a p' | empty.

# $O(n^2) \Rightarrow O(n)$ : source parser

#### Complexity: O(n<sup>2</sup>)

data Symbol = A | B; data List a = Nil | Cons a (List a); data Option a = Some a | None;

#### match doublea word where

match = \p i -> p (eof return) i; return = \x -> Some x; doublea = or nil (join a (join doublea a)); or = \p1 p2 next w -> case p1 next w of { Some w1 -> Some w1; None -> p2 next w;}; nil = \next w -> next w; join = \p1 p2 next w -> p1 (p2 next) w; a = \next w -> case w of { Nil -> None; Cons s w1 -> case s of { A -> next w1; B -> None;};}; b = \next w -> case w of { Nil -> None; Cons s w1 -> case s of { A -> None; B -> next w1;};}; eof = \next w -> case w of { Cons s w1 -> next Nil;};

# $O(n^2) \Rightarrow O(n)$ : residual parser

#### Ordinary SC, complexity $O(n^2)$

```
case word of {
 Cons y9 t5 \rightarrow
  case word of { Cons w13 w9 ->
     case w13 of {
      A -> (letrec f=(r21-> (s21-> case r21 of { Cons r3 y5 ->
       case r3 of { A \rightarrow case (s21 y5) of { Some z7 \rightarrow (Some z7);
                  None ->
                   ((f y 5))
                    (\s8->
                      case s8 of {
                 Cons z5 s18 -> case z5 of { A -> (s21 s18); B -> None; };
                       Nil -> None:
                      }));
                 };
               B \rightarrow None:
              }; Nil -> None;}))
       in
         ((f w9) (v16 -> case v16 of { Cons t6 w2 -> None; Nil -> (Some Nil); })));
      B \rightarrow None:
     \}; Nil \rightarrow None;
  };Nil -> (Some Nil);
}
```

# $O(n^2) \Rightarrow O(n)$ : residual parser

#### 2-level SC, complexity O(n)

```
letrec
f=(\s14->
    case s14 of {
    Cons z12 y8 ->
        case z12 of {
            A -> case y8 of {
            Cons s3 s2 -> case s3 of { A -> (f s2); B -> None; };
            Nil -> None; };
            B -> None; };
            Nil -> (Some Nil);
        }
in
            f word
```

#### Supercompilation relation

 $e' = SC[e] \Rightarrow e SC_{rel} e'$ 

- Andrei V. Klimov. A Program Specialization Relation Based on Supercompilation and its Properties. In First International Workshop on Metacomputation in Russia (Proceedings of the first International Workshop on Metacomputation in Russia. Pereslavl-Zalessky, Russia, July 2-5, 2008). A. P. Nemytykh, Ed. - Pereslavl-Zalessky: Ailamazyan University of Pereslavl, 2008, 108 p. ISBN 978-5-901795-12-5, pages 54-77. <u>http://pat.keldysh.ru/~anklimov/</u>
- Ilya Klyuchnikov. Supercompiler HOSC: proof of correctness. Preprint 31. Keldysh Institute of Applied Mathematics, Moscow, 2010. <u>http://pat.keldysh.ru/~ilya/</u>

The purpose: theoretical (proofs of correctness). Klyuchnikov:  $HOSC_0 \supseteq HOSC_{1/2} \supseteq HOSC$ .

#### Deterministic vs. nondeterministic SC

A taxonomy of supercompilation

#### • Deterministic SC:

e' = SC[e] an operation (a single e').

#### • Nondeterministic SC:

e SC e'

a *relation* (one or more e').

#### • Multi-result SC:

 $MSC[e] \subseteq \{ e' | e SC e' \} \& MSC[e] \neq \emptyset$ an *operation* (a non-empty set of residual programs).

For practical purposes, it is desirable for *MSC*[*e*] to be *finite*.

#### From determinism to non-determinism

Non-determinism is popular in the field of *model-checking*.

A model is produced by throwing away "irrelevant" details.

A typical situation:

if p then e1 else e2

By abstracting away the condition p, we get

**choice** { *e1*; *e2*; }

A tree of possible states at run-time (instead of a sequence).

#### Nondeterministic SC

The supercompilation relation SC can be formulated as a nondeterministic program:

```
t = e0
while incomplete(t) do
  beta = unprocessedLeaf(t)
  t = choice{
      drive(t, beta);
      generalize(t, beta);
      fold(t, beta);
      fail; }
```

end

In this way we abstract away *the whistle* and *the strategies*. But they come back in a multi-result *MSC*!

#### Problems with MSC

- How to make *MSC*[*e*] finite?
  - An answer: by killing partial process trees that make a whistle blow.
- How to (automatically) reduce the size of MSC[e]?
  - An answer: by "normalizing" residual programs and mergin ones with "insignificant" differences.
  - An answer: by analyzing residual programs and throwing away ones that are "dull" and/or "uninteresting".

MSC **opens** a new area of research (rather than gives a "final solution" to a problem).

A simple implementation of MSC (the branch "multi"): <u>https://github.com/ilya-klyuchnikov/spsc-lite-scala</u>

# Proving equivalences by means of *MSC*

Let  $e' \in MSC[e] \Rightarrow e' \cong e$ , i.e. a multi-result supercompiler *MSC* is semantics-preserving.

**Proof technique:** 

$$\exists e' \in MSC[e_1] \cap MSC[e_2] \Leftrightarrow e_1 \cong e_2$$

**Justification** (close to a tautology):

$$e' \in MSC[e_1] \cap MSC[e_2] \Leftrightarrow$$
$$e' \in MSC[e_1] \& e' \in MSC[e_2] \Leftrightarrow$$
$$e' \cong e_1 \& e' \cong e_2 \Leftrightarrow e_1 \cong e_2$$

#### Proving equivalences by transitivity

**The principle**:  $e_1 \cong e_2 \otimes e_2 \cong e_3 \oplus e_1 \cong e_3$ 

#### **Proof technique**:

- Suppose we have failed to proof  $e_1 \cong e_3$ .
- Let us pick up an expression  $e_2$ .
- Suppose we are able to prove both  $e_1 \cong e_2$  and  $e_2 \cong e_3$
- Then we conclude that  $e_1 \cong e_3$ .

Le us check ≅ by means of *MSC*!

# How to use *MSC* for proving equivalences by transitivity?

#### An implementation:

- Suppose  $MSC[e_1] \cap MSC[e_3] = \emptyset$ .
- Let us pick up an expression  $e_2$ .
- If there are

 $e' \in MSC[e_1] \cap MSC[e_2]$  $e'' \in MSC[e_2] \cap MSC[e_3],$ 



A deterministic SC is unable to prove  $e_1 \cong e_3$ , if  $SC[e_1] \equiv SC[e_3]!$ 



Just a speculation. No interesting examples yet. :-( But they are bound to be found by our new postgraduates. :-)

#### MSC and 2-level supercompilation

Some interesting possibilities:

- MSC instead of SC at the lower level.
   More lemmas can be found.
- MSC instead of SC at the upper level.
  - Several different lemmas can be tried at the same node.

**An idea.** Residual programs can be ranked according to their "non-triviality".

 The more improvement lemmas have been applied during 2-level supercompilation, the less trivial is the residual program!

#### MRSC: a framework for creating multiresult supercompilers

 Ilya Klyuchnikov and Sergei Romanenko. Multi-Result Supercompilation as Branching Growth of the Penultimate Level in Metasystem Transitions. Accepted for Ershov Informatics Conference 2011. <u>http://pat.keldysh.ru/~ilya/</u>

Parameterized over

- the object language;
- the language of configurations;
- driving, whistle, generalization.

Provides a number of combinators to produce multi-result and two-level supercompilers from ordinary ones.

### Conclusions

- Supercompilation can be treated as a "primitive operation" in order to build more complex system. This is an instance of "metasystem transition" (in terms of V.F. Turchin).
- By abstracting away the whistle and the strategies, we get nondetermistic supercompilation (a supercompilation relation).
- By "rehabilitating" the whistle and the strategies, we remove some nondeterminism to come to multi-result supercompilation (MSC).
- MSC is more powerful in solving certain problems than deterministic one.
- MSC is a new area of research. Not much is done yet...

#### Thank you!